Proof of Caccetta-Häggkvist conjecture for oriented graphs of independence number 2

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Abstract

In his paper "On the Caccetta-Häggkvist conjecture with forbidden subgraphs" (see [1]), A. Razborov says that Chudnovski and Seymour proved that an out-regular oriented graph of out-degree $d \geq 2$, of independence number 2 and of order at most $3d$ contains a directed triangle, which means that Caccetta-Häggkvist conjecture is verified by such an oriented graph. He says also that to the best of his knowledge, the question is still open without the restriction of out-regularity. In this paper, we give a complete answer, by proving that for $d \geq 2$, any oriented graph of minimum out-degree $d \geq 2$, of independence number 2 and of order at most $3d$ contains a directed triangle. Additionally, we prove that any oriented graph of minimum out-degree $d \geq 1$, of independence number 2 and of order at most $4d$ contains a directed cycle of length at most 4. A simple observation on the girth of a non-acyclic oriented graph of independence number 2, allows us to state that the Caccetta-Häggkvist conjecture is true for oriented graphs of minimum out-degree at least 1 and of independence number 2.

References

[1] A. Razborov, On the Caccetta-Häggkvist conjecture with forbidden subgraphs, submitted to Journ. of Grph Theory (arXiv :1107.2247v1, 2011)